

Analog Image

2, 3, 5
8, 9, 10

Digital Image

- pixel is the main element
- two-dimensional function $f(x, y)$
- Process digital image by means of computer, it cover low, high level process.

DIP 129

Image Acquisition Process

0 → black
255 → white.

formation model $f(x,y) = i(x,y) \cdot r(x,y)$

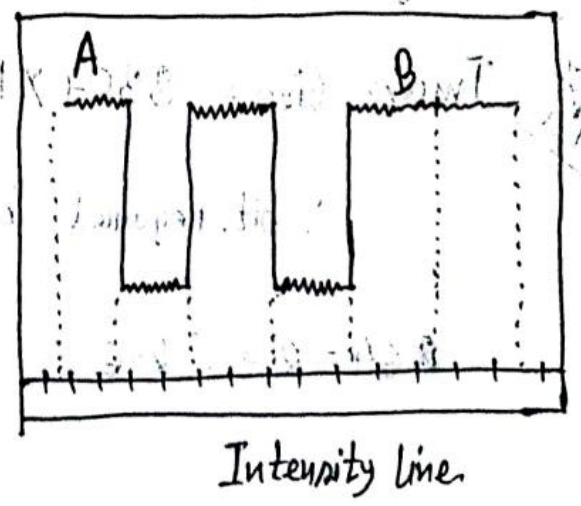
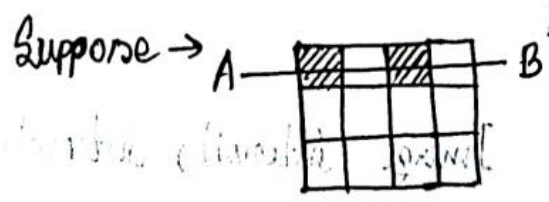
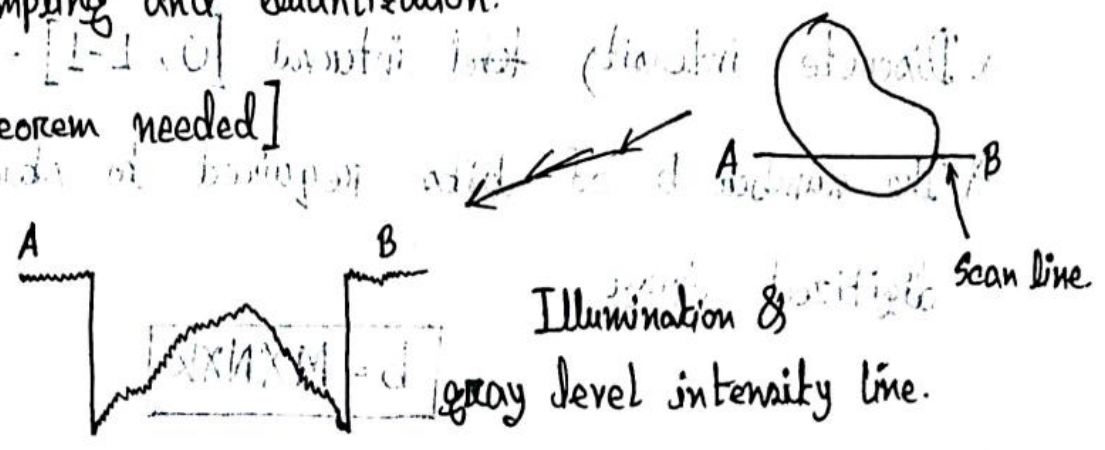
intensity at the point (x,y)

illumination

Reflectance
Range (0-1)

Image Sampling and Quantization.

[Nyquist theorem needed]



Draw the intensity line from the scan line AB.

Question

☐ Image sampling: ~~digitize~~ of x-axis digitalization

☐ Quantization: Digitalize of y-axis.

☐ Representing Digital Image: $M \times N$ numerical array

matrix.

$$f(x, y) = \begin{bmatrix} f(0,0), & f(0,1) & \dots & f(0,n-1) \\ \vdots & \vdots & & \vdots \\ f(m-1,0), & \dots & \dots & f(m-1,n-1) \end{bmatrix}$$

* Discrete intensity level interval $[0, L-1]$, $L = 2^k$

* The number b of bits required to store a $M \times N$ digitized image.

$$b = M \times N \times k$$

~~Image~~ Image Resolution = $M \times N$.

Image Size = 3304×1640 ; Image intensity interval = $0-63$.

\therefore bit required to store $b = M \times N \times k$

$$= 3304 \times 1640 \times 6$$

$$= 32511360 \text{ bits.}$$

264 = 2^6 $\therefore k=6$

★
Question

Spatial and Intensity Resolution

→ A measure of the resolution

$$(D)_{eff} \leftarrow \text{to analyze length}$$

$$(I)_{eff} \leftarrow \text{to analyze intensity}$$

$$(D)_{eff} = (D)_{eff} \cup (I)_{eff} \leftarrow \text{to analyze } \lambda \neq$$

no vector along λ axis \leftarrow \neq \neq

no vector along ρ axis of oblique axis \leftarrow \neq \neq

no $(I)_{eff}$ for all ρ \neq \neq

no $(D)_{eff}$ for all λ \neq \neq

no vector along ρ axis

2-10

Basic Relationships between Pixels

Neighbors of a pixel p at coordinates (x, y)

4 neighbors of p , $\rightarrow N_4(p)$

4 diagonal neighbors of $p \rightarrow N_D(p)$

8 neighbors of $p \rightarrow N_4(p) \cup N_D(p) = N_8(p)$

Adjacency - 4 \rightarrow Let V be the set of intensity values.

m-adjacency: Two pixels p and q with values from V are m-adjacency if,

(i) q is in the set $N_4(p)$, or

(ii) q is in the set $N_D(p)$ and the set $N_4(p) \cap N_4(q)$

has no pixels whose values are from V .

★ Question Finding m-adjacency where $V = \{1, 2, 3\}$

$P, Q = (0, 1), (1, 1)$

$P_2, Q_2 = (1, 1), (2, 2)$

$P_3, Q_3 = (2, 3), (3, 3)$

0	1	0	0
3	2	0	0
0	0	1	③ ^P
2	0	0	① ^Q

here, $P = (0, 1) \rightarrow 1$

$Q = (1, 1) \rightarrow 2$

0	① ^P	0	0
3	② ^Q	0	0
0	0	1	3
2	0	0	1

for 1P, 2Q it fulfill the first condition

Q is in the set $N_4(P)$

So, they are m-adjacency.

here, $P = (1, 1) \rightarrow 2$

$Q = (2, 2) \rightarrow 1$

0	1	0	0
3	② ^P	0	0
0	0	① ^Q	3
2	0	1	1

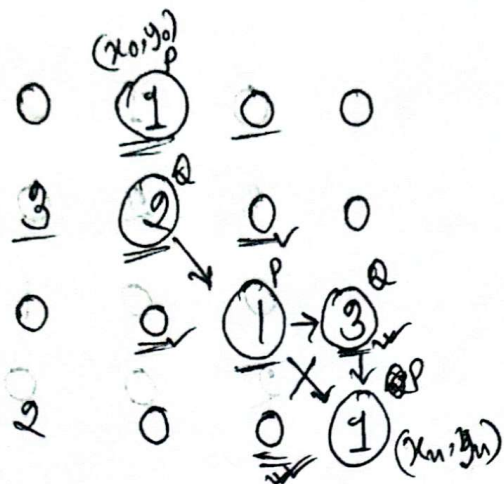
here the $N_4(P) \cap N_4(Q) = \{(0, 0)\}$

and no pixels is in values of V,

So they are m-adjacency.

Question

Is it possible to find the path using m-adjacency here



then find the path

Connected in S

$(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$

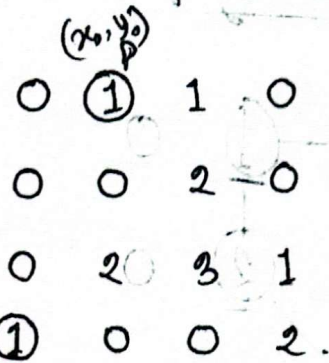
where $\forall i, 0 \leq i \leq n, (x_i, y_i) \in S$

PS

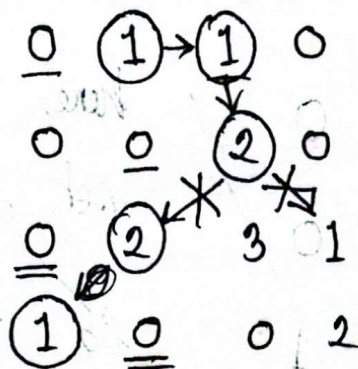
Recursion

Find 4-adjacency, 8-adjacency and m-adjacency where path.

$V = \{1, 2\}$



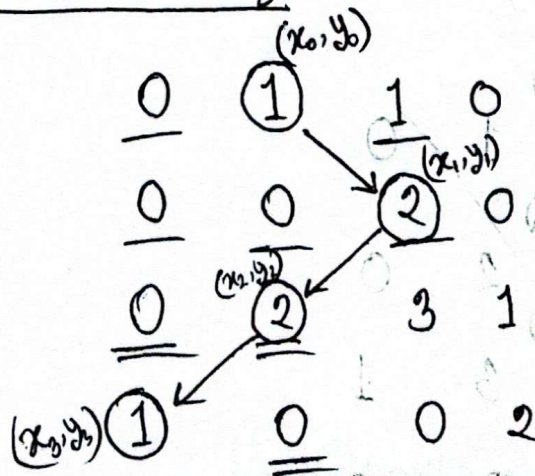
finding 4-adjacency



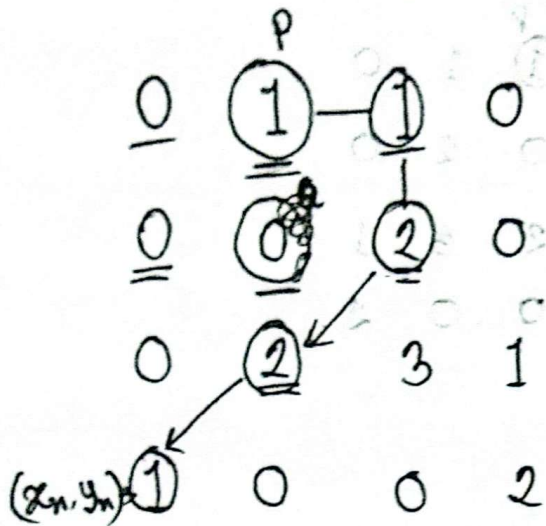
Not possible

No path with 4-adjacency.

finding 8-adjacency

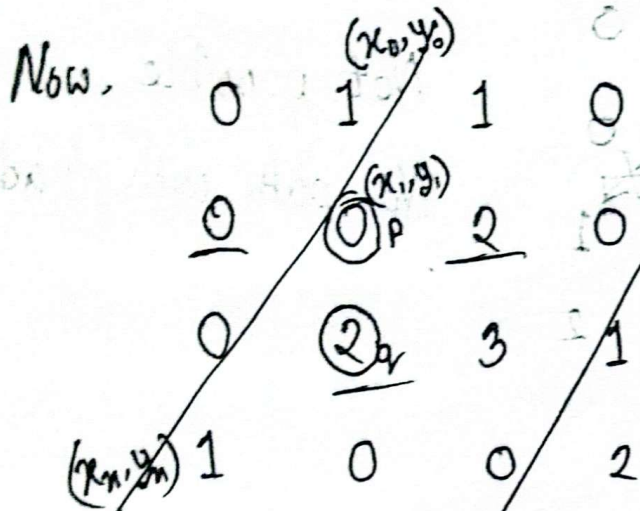


Finding m-adjacency: $V = \{1, 2\}$



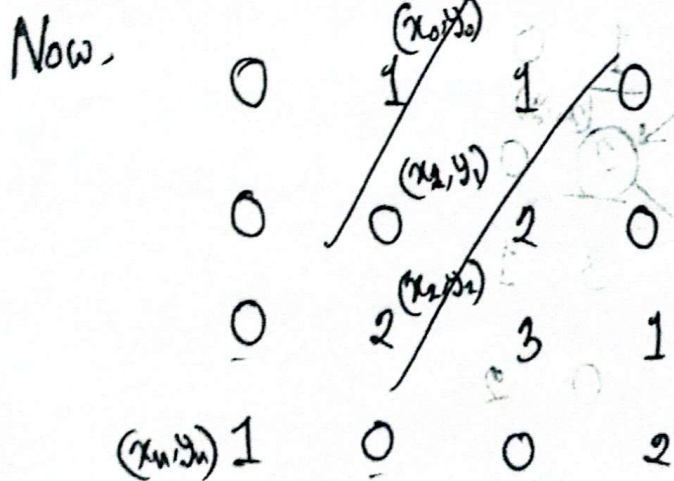
here, 2 is in the set of $N_4(P)$

So, they are m-adjacent



here, 2 is in the set of $N_4(P)$

So ✓



Boundary (or border)

↳ The boundary of a region R is the set of pixels in the region that have one or more neighbors that are not in R .

↳ If R happens to be an entire image, then its boundary is defined as the set of pixels in the first and last rows and columns of the image.

Foreground and background

↳ An image contains k disjoint regions, (R_k) ,

$k = 1, 2, 3, \dots, k$. Let R_u denote the union of all the k regions. Let $(R_u)^c$ denote its complement.

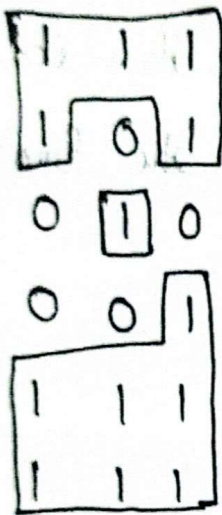
★ Slide page - 67, 68 Question

(Answer 10) problem 11

1. Part 1, 2 is adjacent using 4-ad? No

2. How many regions are here? 3

3. What is the foreground and background? $\overset{F}{1}, \overset{B}{0}$



Boundary

→ কোনো Region এর কমপক্ষে

৪টি Neighbors বাইরে থাকবে.

৪-এর 4 adjacency দেখিয়ে করতে

হবে.

Distance Measures:

Given pixels p, q and z with coordinates $(x, y), (s, t), (u, v)$ respectively, the distance function D has following properties:

- a) $D(p, q) \geq 0$ [$D(p, q) = 0$, iff $p = q$] iff (if and only if)
- b) $D(p, q) = D(q, p)$
- c) $D(p, z) \leq D(p, q) + D(q, z)$

Distance measuring questions ***

Three types of distance measurement method.

a) Euclidean distance: $D_e(p, q) = [(x-s)^2 + (y-t)^2]^{\frac{1}{2}}$

b) City Block distance: $D_4(p, q) = |x-s| + |y-t|$

		2		
	2	1	2	
2	1	0	1	2
	2	1	2	
		2		

c) Chess Board distance: $D_8(p, q) = \max(|x-s|, |y-t|)$

$P = (3, 3), Q = (4, 5)$

$$D_8(p, q) = \max(|3-4|, |3-5|)$$

$$= \max(1, 2)$$

$$= 2$$

2	2	2	1	2
2	1	1	1	2
2	1	0	1	2
2	1	1	1	2
2	2	2	2	2

Quest:

0 0 0 0 0

0 0 1 ① 0

0 1 1 0 0

0 ① 0 0 0

0 0 0 0 0

0 0 0 0 0

Suppose,

Origin is in lower left.

So, $P = (2, 3)$

$Q = (4, 5)$

using chess board distance,

$$D_8(P, Q) = \max(|x_1 - x_2|, |y_1 - y_2|)$$

$$= \max(|2 - 4|, |3 - 5|)$$

$$= \max(2, 2)$$

$$|x - y| + |z - x| = 2$$

using city block distance,

$$D_4(P, Q) = |x - s| + |y - t|$$

$$= 2 + 2$$

$$= 4$$

0	0	0	0	0
0	0	1	①	0
0	1	1	0	0
0	①	0	0	0
0	0	0	0	0

Linear and Nonlinear Operation:

$$\mathcal{H}[f(x,y)] = g(x,y)$$

Now,

$$\begin{aligned} \mathcal{H}[af_1(x,y) + bf_2(x,y)] &= a\mathcal{H}[f_1(x,y)] + b\mathcal{H}[f_2(x,y)] \\ &= ag_1(x,y) + bg_2(x,y) \end{aligned}$$

Suppose $a=1$, $b=-1$

$$f_1(x,y) = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}, \quad f_2(x,y) = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$$

$$\begin{aligned} \Sigma [af_1(x,y) + bf_2(x,y)] & \quad \left| \quad a\Sigma f_1(x,y) + b\Sigma f_2(x,y) \right. \\ = \Sigma \left[(1) \cdot \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1) \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \right] & = 1 \Sigma \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1) \Sigma \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix} \\ = \Sigma \left[\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + \begin{bmatrix} -4 & -5 \\ -6 & -7 \end{bmatrix} \right] & = 1 \cdot 7 + (-1) \cdot 22 \\ = \Sigma \begin{bmatrix} -4 & -3 \\ -4 & -4 \end{bmatrix} & = -15 \\ = -15 & \end{aligned}$$

So, the operation Σ is a linear operator.

Arithmetic Operation!

Used in noise removal.

$$\text{Noiseless image} = f(x, y)$$

$$\text{Noise} = n(x, y)$$

$$\text{Corrupt image} = g(x, y)$$

$$\text{So, } g(x, y) = f(x, y) + n(x, y)$$

Reducing noise by adding a set of noisy images $\{g_i(x, y)\}$

$$\bar{g}(x, y) = \frac{1}{k} \sum_{i=1}^k g_i(x, y)$$

Ques! How addition of Noise can be used for noise removal or noise reduction

explain with example

Ques 2! noise reduce math

Set and Logical Operation

Let A be the elements of a gray-scale image.

$A = \{(x, y, z) \mid z = f(x, y)\}$ z is intensity at the point (x, y)

The complement of A is denoted as A^c

$A^c = \{(x, y, k-z) \mid (x, y, z) \in A\}$

Ques: find comp of A . $A^c = ?$
 $z = ?$

$k = 2^k - 1$; k is the number of intensity bits used to represent z .

* Not, And, AndNot, XOR operations.

Intensity Transformation and Spatial Filtering

* Spatial Domain: Image plane, directly process the intensity values of the image plane.

* Transform Domain:

Spatial Domain Process

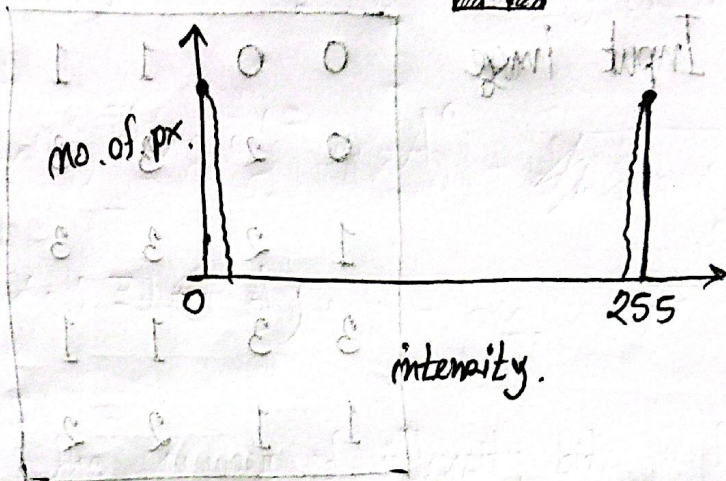
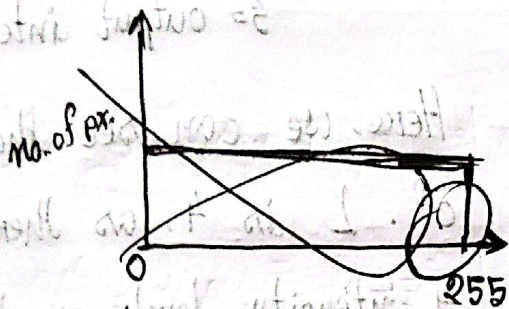
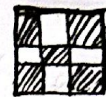
$T \rightarrow$ intensity value transform

T : an operator

Intensity Transformation Function:

0 → black
255 → white

histogram of chess board:



for a white page:

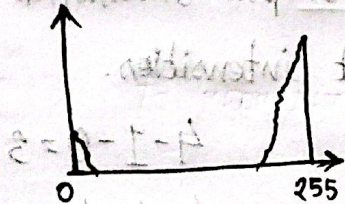


Image Negative - $s = (L-1) - r$

\downarrow \downarrow
 255 255

Intensity Levels Question -

0	0	0	0
0	0	1	0
0	0	1	0
0	0	0	0
1	1	0	0

Input	Output
0	255
1	254
0	255
0	255

Image Negative - $S = L - 1 - p$ [We know]

↓
Level

where,

$p =$ input intensity

$S =$ output intensity

Input image

0	0	1	1
0	2	3	3
1	2	3	3
3	3	1	1
1	1	2	2

Here, we can see the value of L is 4, as there are 4 intensity levels on the provided image. Now, calculating the output intensities from the input intensities.

Output Image -

3	3	2	2
3	1	0	0

$$4 - 1 - 0 = 3$$

$$4 - 1 - 1 = 2$$

$$4 - 1 - 2 = 1$$

input	Output
0	$S = 4 - 1 - 0 = 3$
1	$S = 4 - 1 - 1 = 2$
2	$S = 4 - 1 - 2 = 1$
3	$S = 4 - 1 - 3 = 0$

So, after replacing the pixel values with the output intensities, we got,

3	3	2	2
3	1	0	0
2	1	0	0
0	0	2	2
2	2	1	1

□ Log Transformation:

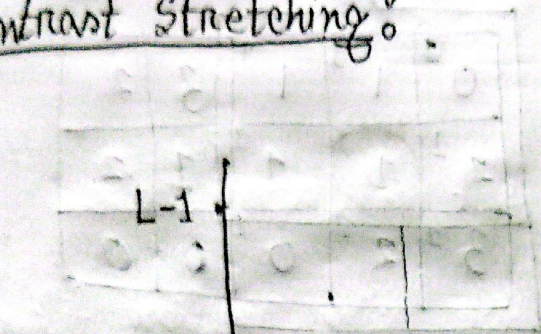
$$S = c \log(1+h)$$

where, $c = \text{Constant}$

□ Power ~~Law~~ (GAMMA) Transformation:

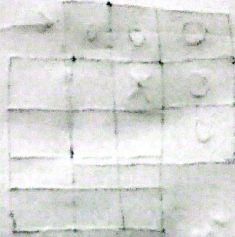
$$S = c r^\gamma$$

Contrast Stretching:

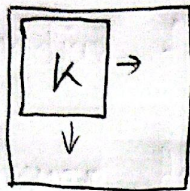


(r, s)

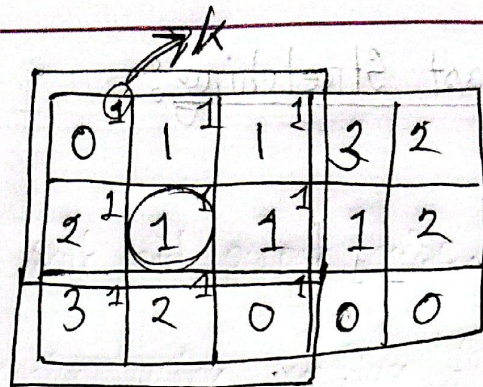
p_1



Spatial Filtering:

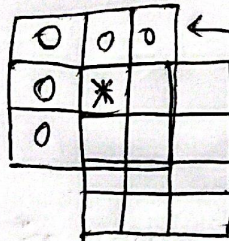


k = kernel



$$0 \cdot 1 + 1 \cdot 1 + 1 \cdot 1 + 2 \cdot 1 + 1 \cdot 1 + 3 \cdot 1 + 2 \cdot 1 + 0 \cdot 1 = 15$$

Kernel Padding:



$$\frac{15}{3} = 5$$

Correlation & Convolution:

~~Question~~

	★ f	★ W	Result
(a)	0 0 0 1 0 0 0 0	1 2 4 2 8	
(b)	$\begin{matrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \times 1 & \times 2.0 & \times 4.0 & \times 2.0 & \times 8.0 & & & & & & \\ 1 & 2 & 4 & 2 & 8 & = 1 \cdot 0 + 2 \cdot 0 + 4 \cdot 0 + 2 \cdot 0 + 8 \cdot 0 = & & & & & \end{matrix}$		0
(c)	1 2 4 2 8		= 8
(d)	1 2 4 2 8		= 2
(e)	1 2 4 2 8		= 4
(f)	1 2 4 2 8		= 2
(g)	1 2 4 2 8		= 1

Similarly after 2 more shift = 0 8 2 4 2 1 0 0

Convolution is similar to correlation But we have to rotate ω 180° .

Box filter kernel:

Using low pass / ~~smoothing~~ kernel reduce noise

Suppose a noisy image =

10	11	10	13	10
15	<u>90</u>	<u>75</u>	17	12
10	11	<u>89</u>	16	15
12	13	10	11	12
10	15	17	16	16

So, for 75 =

$$\frac{1}{9} \times [90 + 75 + 17 + 11 + 10 + 13 + 11 + 89 + 16]$$

$$= 36.89 \approx 37$$

0 1 0
0 1 1
0 1 0

— low pass kernel

Linear & Nonlinear filters:

* median filters can remove salt and pepper noise.

* Remove the salt and pepper noise using median filter.

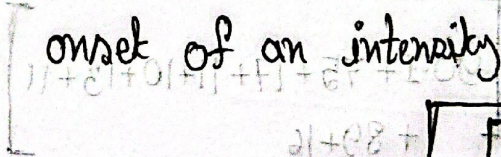
Derivative:

01	21	01	11	01
21	21	22	11	01
21	21	22	11	01

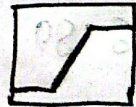
1. Must be zero in areas of constant intensity.



2. Must be nonzero at the onset of an intensity step or ramp



3. Must be nonzero along intensity ramp



We will get sharp photo on 2nd derivative.

Laplacian kernel —

0	1	0
1	-4	0
0	1	0

Summation must be zero.

* Practice: Box Filter kernel :-

Suppose a noisy image -

change the value of noise pixel.

10	12	13	10	10
11	12	30	12	10
15	14	13	18	12
30	35	32	51	50
50	52	52	50	50
53	53	50	50	51

So, for 12 -

$$= \frac{1}{9} (10+12+13+11+12+30+15+14+13)$$

$$= 14$$

Now 12 will replace with 14

This is how the pixel will replace